

Avatars of the Tortoise

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There is a concept which corrupts and upsets all others. I refer not to Evil, whose limited realm is that of ethics; I refer to the infinite. I once longed to compile its mobile history. The numerous Hydra (the swamp monster which amounts to a prefiguration or emblem of geometric progressions) would lend convenient horror to its portico; it would be crowned by the sordid nightmares of Kafka and its central chapters would not ignore the conjectures of that remote German cardinal – Nicholas of Krebs, Nicholas of Cusa – who saw in the circumference of the circle a polygon with an infinite number of sides and wrote that an infinite line would be a straight line, a triangle, a circle and a sphere (*De docta ignorantia*, I, 13). Five or seven years of metaphysical, theological and mathematical apprenticeship would allow me (perhaps) to plan decorously such a book. It is useless to add that life forbids me that hope and even that adverb.

The following pages in some way belong to that illusory *Biography of the Infinite*. Their purpose is to register certain avatars of the second paradox of Zeno.

Let us recall, now, that paradox.

Achilles runs ten times faster than the tortoise and gives the animal a headstart of ten meters. Achilles runs those ten meters, the tortoise one; Achilles runs that meter, the tortoise runs a decimeter; Achilles runs that decimeter, the tortoise runs a centimeter; Achilles runs that centimeter, the tortoise, a millimeter; Fleet-footed Achilles, the millimeter, the tortoise, a tenth of a millimeter, and so on to infinity, without the tortoise ever being overtaken. . . Such is the customary version. Wilhelm Capelle (*Die Vorsokratiker*, 1935, page 178) translates the original text by Aristotle: “The second argument of Zeno is the one known by the name of Achilles. He reasons that the slowest will never be overtaken by the swiftest, since the pursuer has to pass through the place the pursued has just left, so that the slowest will always have a certain advantage.” The problem does not change, as you can see; but I would like to know the name of the poet who provided it with a hero and a tortoise. To those magical competitors and to the series

$$10 + 1 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10,000} + \dots$$

the argument owes its fame. Almost no one recalls the one preceding it – the one about the track –, though its mechanism is identical. Movement is impossible (argues Zeno) for the moving object must cover half of the distance in order to reach its destination, and before reaching the half, half of the half, and before half of the half, half of the half of the half, and before. . .¹

We owe to the pen of Aristotle the communication and first refutation of these arguments. He refutes them with a perhaps disdainful brevity, but their recollection served as an inspiration for his famous *argument of the third man* against the Platonic doctrine. This doctrine tries to demonstrate that two individuals who have common attributes (for example, two men) are mere temporal appearances of an eternal archetype. Aristotle asks if the many men and the Man – the temporal individuals and the archetype – have attributes in common. It is obvious that they do: the general attributes of humanity. In that case, maintains Aristotle, one would have to postulate *another* archetype to include them all, and then a fourth. . . Patricio de Azcárate, in a note to his translation of the *Metaphysics*, attributes this presentation of the

¹A century later, the Chinese sophist Hui Tzu reasoned that a staff cut in two every day is interminable (H. A. Giles: *Chuang Tzu*, 1889, page 453).

problem to one of Aristotle's disciples: "If what is affirmed of many things is at the same time a separate being, different from the things about which the affirmation is made (and this is what the Platonists pretend), it is necessary that there be a third man. *Man* is a denomination applicable to individuals and the idea. There is, then, a third man separate and different from individual men and the idea. There is at the same time a fourth man who stands in the same relationship to the third and to the idea and individual men; then a fifth and so on to infinity." Let us postulate two individuals, *a* and *b*, who make up the generic type *c*. We would then have:

$$a + b = c.$$

But also, according to Aristotle:

$$\begin{aligned} a + b + c &= d \\ a + b + c + d &= e \\ a + b + c + d + e &= f \dots \end{aligned}$$

Rigorously speaking, two individuals are not necessary: it is enough to have one individual and the generic type in order to determine the *third man* denounced by Aristotle. Zeno of Elea resorts to the idea of infinite regression against movement and number; his refuter, against the idea of universal forms.²

The next avatar of Zeno my disorderly notes register is Agrippa the skeptic. He denies that anything can be proven, since every proof requires a previous proof (*Hypotyposes*, I, 166). Sextus Empiricus argues in a parallel manner that definitions are in vain, since one will have to define each of the words used and then define the definition (*Hypotyposes*, II, 207). One thousand six hundred years later, Byron, in the dedication to *Don Juan*, will write of Coleridge: "I wish he would explain his Explanation."

So far, the *regressus in infinitum* has served to negate; Saint Thomas Aquinas resorts to it (*Summa theologica*, I, 2, 3) in order to affirm that God exists. He points out that there is nothing in the universe without an effective cause and that this cause, of course, is the effect of another prior cause. The world is an interminable chain of causes and each cause is also an effect. Each state derives from a previous one and determines the following, but the whole series could have not existed, since its terms are conditional, i.e., fortuitous. However, the world *does* exist; from this we may infer a noncontingent first cause, which would be the Divinity. Such is the cosmological proof; it is prefigured by Aristotle and Plato; later Leibniz rediscovers it.³

²In the *Parmenides* – whose Zenonian character is irrefutable – Plato expounds a very similar argument to demonstrate that the one is really many. If the one exists, it participates in being; therefore, there are two parts in it, which are being and the one, but each of these parts is one and exists, so that they enclose two more parts, which in turn enclose two more, infinitely. Russell (*Introduction to Mathematical Philosophy*, 1919, page 138) substitutes for Plato's geometrical progression an arithmetical one. If one exists, it participates in being; but since being and the one are different, duality exists; but since being and two are different, trinity exists, etc. Chuang Tzu (Waley: *Three Ways of Thought in Ancient China*, page 25) resorts to the same interminable *regressus* against the monists who declared that the Ten Thousand Things (the Universe) are one. In the first place – he argues – cosmic unity and the declaration of that unity are already two things; these two and the declaration of their duality are already three; those three and the declaration of their trinity are already four . . . Russell believes that the vagueness of the term *being* is sufficient to invalidate this reasoning. He adds that numbers do not exist, that they are mere logical fictions.

³An echo of this proof, now defunct, resounds in the first verse of the *Paradiso*:

La gloria di Colui che tutto move.

Hermann Lotze has recourse to the *regressus* in order not to understand that an alteration of object *A* can produce an alteration of object *B*. He reasons that if *A* and *B* are independent, to postulate an influence of *A* on *B* is to postulate a third element *C*, an element which in order to affect *B* will require a fourth element *D*, which cannot work its effect without *E*, which cannot work its effect without *F*. . . In order to elude this multiplication of chimeras, he resolves that in the world there is one sole object: an infinite and absolute substance, comparable to the God of Spinoza. Transitive causes are reduced to immanent causes; phenomena, to manifestations or modalities of the cosmic substance.⁴

Analogous, but even more alarming, is the case of F. H. Bradley. This thinker (*Appearance and Reality*, 1897, pages 19-34) does not limit himself to combatting the relation of cause; he denies all relations. He asks if a relation is related to its terms. The answer is yes and he infers that this amounts to admitting the existence of two other relations, and then of two more. In the axiom “the part is less than the whole” he does not perceive two terms and the relation “less than”; he perceives three (“part,” “less than,” “whole”) whose linking implies two more relations, and so on to infinity. In the statement “John is mortal,” he perceives three invariable concepts (the third is the copula) which we can never bring together. He transforms all concepts into incommunicable, solidified objects. To refute him is to become contaminated with unreality.

Lotze inserts Zeno’s periodic chasms between the cause and the effect; Bradley, between the subject and the predicate, if not between the subject and its attributes; Lewis Carroll (*Mind*, volume four, page 278), between the second premise of the syllogism and the conclusion. He relates an endless dialogue, whose interlocutors are Achilles and the tortoise. Having now reached the end of their interminable race, the two athletes calmly converse about geometry. They study this lucid reasoning:

- (a) Two things equal to a third are equal to one another.
- (b) The two sides of this triangle are equal to *MN*.
- (c) The two sides of this triangle are equal to one another.

The tortoise accepts the premises (a) and (b), but denies that they justify the conclusion. He has Achilles interpolate a hypothetical proposition:

- (a) Two things equal to a third are equal to one another.
- (b) The two sides of this triangle are equal to *MN*.
- (c) If (a) and (b) are valid, (*z*) is valid.
- (z) The two sides of this triangle are equal to one another.

Having made this brief clarification, the tortoise accepts the validity of (a), (b) and (c), but not of (z). Achilles, indignant, interpolates:

- (d) if (a), (b) and (c) are valid, (*z*) is valid.

And then, now with a certain resignation:

- (e) if (a), (b) (c), and (d) are valid, (*z*) is valid.

⁴I follow the exposition by James (*A Pluralistic Universe*, 1909, pages 55-60). Cf. Wentscher: *Fechner und Lotze*, 1924, pages 166-171.

Carroll observes that the Greek's paradox involves an infinite series of distances which diminish, whereas in his, the distances grow.

One final example, perhaps the most elegant of all, but also the one differing least from Zeno. William James (*Some Problems of Philosophy*, 1911, page 182) denies that fourteen minutes can pass, because first it is necessary for seven to pass, and before the seven, three and a half, and before the three and a half, a minute and three quarters, and so on until the end, the invisible end, through tenuous labyrinths of time.

Descartes, Hobbes, Leibniz, Mill, Renouvier, Georg Cantor, Gomperz, Russell and Bergson have formulated explanations – not always inexplicable and vain in nature – of the paradox of the tortoise. (I have registered some of them in my book *Discusión*, 1932, pages 151-161). Applications abound as well, as the reader has seen. The historical applications do not exhaust its possibilities: the vertiginous *regressus in infinitum* is perhaps applicable to all subjects. To aesthetics: such and such a verse moves us for such and such a reason, such and such a reason for such and such a reason. . . . To the problem of knowledge: cognition is recognition, but it is necessary to have known in order to recognize, but cognition is recognition. . . . How can we evaluate this dialectic? Is it a legitimate instrument of investigation or only a bad habit?

It is venturesome to think that a coordination of words (philosophies are nothing more than that) can resemble the universe very much. It is also venturesome to think that of all these illustrious coordinations, one of them – at least in an infinitesimal way – does not resemble the universe a bit more than the others. I have examined those which enjoy certain prestige; I venture to affirm that only in the one formulated by Schopenhauer have I recognized some trait of the universe. According to this doctrine, the world is a fabrication of the will. Art – always – requires visible unrealities. Let it suffice for me to mention one: the metaphorical or numerous or carefully accidental diction of the interlocutors in a drama. . . . Let us admit what all idealists admit: the hallucinatory nature of the world. Let us do what no idealist has done: seek unrealities which confirm that nature. We shall find them, I believe, in the antinomies of Kant and in the dialectic of Zeno.

“The greatest magician (Novalis has memorably written) would be the one who would cast over himself a spell so complete that he would take his own phantasmagorias as autonomous appearances. Would not this be our case?” I conjecture that this is so. We (the undivided divinity operating within us) have dreamt the world. We have dreamt it as firm, mysterious, visible, ubiquitous in space and durable in time; but in its architecture we have allowed tenuous and eternal crevices of unreason which tell us it is false.

Translated by J. E. I.

The next of Zeno's avatars which my disorderly notes register is Agrippa, the skeptic. He denies that something can be proved, for all proof requires a previous proof (Hypotyposes, I, 166). Sextus Empiricus argues similarly that definitions are vain, as one would need to define each of the words used and, then, define the definition (Hypotyposes, II, 207).¹ The tortoise accepts the premises a and b, but denies that they justify the conclusion. He gets Achilles to insert a hypothetical proposition. a) Things that are equal to the same are equal to each other. b) The two sides of this triangle are things that equal to the same. c) If a and b are true, z is true. z) The two sides of this triangle are equal to each other.