

H. F. TIERSTEN AND CONTINUUM ELECTRODYNAMICS

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ABSTRACT: Harry F. Tiersten (1930-2006), Professor of Mechanics at Rensselaer Polytechnic Institute, passed away suddenly on June 12, 2006 from a heart attack.^[1] Professor Tiersten was one of the founders of continuum electrodynamics. In this paper we present a brief summary of Tiersten's major contributions to the theories of continuum electrodynamics and their applications.

I. BRIEF BIOGRAPHY

Professor Tiersten was born in 1930. He obtained his B.S., M.S., and Ph.D. in 1952, 1956, and 1961 respectively, all from Columbia University. His Ph. D. advisor was Professor Raymond D. Mindlin. From 1952-53 he worked as stress analyst with Grumman Aircraft Engineering Corp., and during 1953-56 as structural designer with J. G. White Engineering Corp (New York City). He was an Instructor in Civil Engineering at the City College of the City of New York, and was a research assistant at Columbia University (1960-61). He Joined Bell Telephone Laboratories from 1961-68, and was a visiting professor at Rensselaer and joined the faculty in 1968. He was a member and fellow of IEEE, Acoustical Society of America, American Society of Mechanical Engineers and the Society of Engineering Science. He was the recipient of the Sawyer award from the IEEE Ultrasonics, Ferroelectrics and Frequency Control Society in 1979 for contributions to the theory of piezoelectric resonators.



Professor Tiersten in his office at RPI, behind a pile of files on his desk.

II. NONLINEAR THEORY OF DEFORMABLE FERROMAGNETICS

The first nonlinear theory Tiersten developed is for deformable ferromagnetic materials.^[2,3] Here we emphasize that Tiersten's theory given in Ref. [2] was formulated in 1964, two years earlier than the most referenced book by Brown on this topic which was published in 1966.^[4] This historical fact is probably not known by many researchers and we take this opportunity to set the record straight.

III. NONLINEAR ELECTROELASTICITY

This is the area in which Tiersten had made a series of fundamental contributions and formed a systematic theoretical structure. It was motivated by technology and serves as a good example for how technology stimulates the development of theory and science. Since the 1950s piezoelectric crystal resonators began to be used as components for telecommunication and control equipment for both civilian and military applications. Piezoelectric resonators are resonant devices operating at a particular frequency which is sensitive to temperature and stress or strain. While this allows resonators to be used for making sensors, for telecommunication and control a stable frequency insensitive to environmental effects is desired. To predict resonant frequencies the linear theory of piezoelectricity is sufficient. However, resonator frequency stability analysis is nonlinear in nature. Tiersten systematically developed the theory of nonlinear electroelasticity,^[5] the linear theory for small fields superposed on finite biasing fields^[6] which is needed for stability analysis and has to be obtained from the nonlinear theory, and the perturbation integral^[7] for frequency shifts in resonators based on the theory in Ref. [6]. The theories in Refs. [5-7] were summarized in Ref. [8]. To analyze nonlinear phenomena like nonlinear resonance, harmonic generation and inter-modulation, a cubic theory including all third-order effects of the displacement gradient and electric potential gradient was derived in Ref. [9].

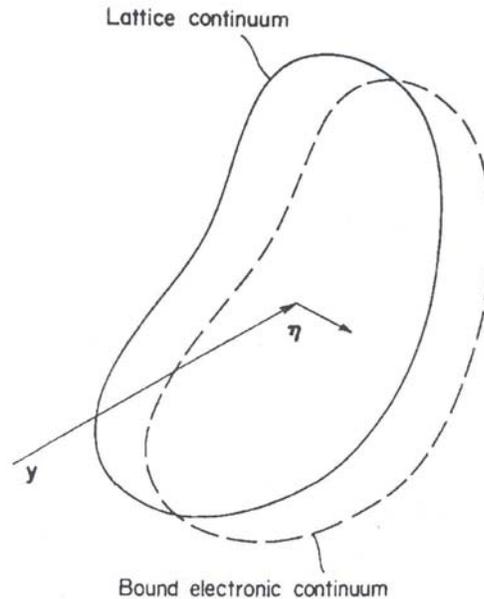


Fig. 1 Interacting and inter-penetrating continua.

Here we point out that according to Professor Tiersten himself, his major contribution is the development of a model using interacting multi-continua to derive equations of continuum

electrodynamics. This method was used in the development of the nonlinear theory of ferromagnetic materials,^[2] electroelastic materials,^[5] electromagnetic-elastic interactions and elastic semiconductors to be discussed below. Take electroelastic materials^[5] as an example, Tiersten introduced a physical model of two mechanically and electrically interacting and interpenetrating continua (see Fig. 1) to describe electric polarization macroscopically. One continuum is the lattice continuum that carries mass and positive charges, the other is the electronic continuum which is negatively charged and is without mass. Electric polarization is modeled by a small, relative displacement of the electronic continuum with respect to the lattice continuum. By systematic applications of the basic laws of physics to each continuum and combining the resulting equations, Tiersten obtained the equations for nonlinear electroelasticity.

IV. NONLINEAR ELECTROMAGNETIC-ELASTIC INTERACTIONS

A very general theory for finitely deformable, polarizable, magnetizable and heat conducting solid continua in interaction with electromagnetic fields was derived in Ref. [10] using a multi-continuum model consisting of an electronic charge and spin continuum and a lattice continuum which in itself consists of two ionic continua which can displace relative to each other to produce ionic polarization. Since spin angular momentum and electronic and ionic linear momentum are taken into account, magnetic spin resonance and both electronic and ionic polarization resonances are included in the treatment.

V. NONLINEAR ELASTIC SEMICONDUCTORS

Many piezoelectric crystals are in fact semiconductors. An acoustic wave propagating in a piezoelectric crystal is usually accompanied by an electric field. When the crystal is also semiconducting, the electric field produces currents and space charge resulting in dispersion and acoustic loss. The interaction between a traveling acoustic wave and mobile charges in piezoelectric semiconductors is called the acoustoelectric effect which is a special case of a more general phenomenon which may be called wave-particle drag. The acoustoelectric effect can also be produced in composites of piezoelectric dielectrics and nonpiezoelectric semiconductors. In these composites the acoustoelectric effect is due to the combination of the piezoelectric effect and the semiconduction in each component phase. It was also found that an acoustic wave traveling in a piezoelectric semiconductor can be amplified by the application of a dc electric field. The acoustoelectric effect and the acoustoelectric amplification of acoustic waves have led to the development of acoustoelectric amplifiers of acoustic waves. In addition, the acoustoelectric effect can also be used to design experiments for measuring charge mobility and make devices for charge transfer driven by acoustic waves. A linear theory for acoustoelectric effect and amplification can be obtained by extending the linear theory of piezoelectricity to include conduction (and diffusion), conservation of charge and linearization about the electric bias.

de Lorenzi and Tiersten^[11] used a macroscopic physical model to derive a nonlinear theory for deformable semiconductors. The model consists of several interacting continua representing the lattice, electrons, and holes, etc. Basic laws of physics are applied to each continuum. The resulting equations are combined to obtain a macroscopic description of the material. Integral forms of the equations were later given in Ref. [12].

VI. A FEW DISSIPATIVE EFFECTS

In the 1990s smart materials and structures was one of the hot areas of research. Tiersten became involved at the request of other researchers because of his reputation on piezoelectricity. In smart structure applications polarized ferroelectric ceramics are often used.

To explain the implications of material objectivity in materials with an initial polarization Tiersten derived nonlinear equations for rate type dissipative effects in electroelastic materials in Ref. [13]. Another type of dissipative effects, hysteresis, is also associated with polarized ceramics. This was treated using internal variables in Ref. [14] nonlinearly. Although piezoelectric materials are treated as dielectrics in the classical theory of piezoelectricity, real materials more or less have a little electrical conduction. This has certain important implications particularly in static and low-frequency problems and is discussed in Ref. [15].

VII. THEORIES OF PIEZOELECTRIC PLATES

In the analysis of piezoelectric plates, whether the major surfaces of a plate are electroded or not makes a significant difference. Some equations developed by various researchers are convenient for one case but not for the other. In obtaining two-dimensional plate equations from the three-dimensional equations it is necessary to introduce some approximation of the fields along the plate thickness. In Ref. [16] Tiersten used an original polynomial expansion of the electric potential in terms of the plate thickness coordinate. In his expansion, except the first two terms, all other terms vanish at the major faces of the plate. As a consequence, the equations obtained are convenient to use for either electroded or unelectroded plates. Later in Ref. [17] he studied the most general case of using an arbitrary expansion of the electric potential and treated the electrodes as electrical constraints. With variational formulation and Lagrange multipliers he made everything about the expansion of the electric potential clear for both unelectroded and electroded plates.

VIII. ANALYSIS OF PIEZOELECTRIC RESONATORS

For the frequency analysis of plate resonator with general material anisotropy within the linear theory of piezoelectricity, the mechanical displacement vector was decomposed along the eigenvectors of pure plate thickness modes. A single-mode equation was obtained for thickness-shear modes, the operating modes of resonators, with slow in-plane variations.^[18] This equation is simple and has led to many very useful solutions.

Tiersten's work on frequency stability of piezoelectric resonators was supported by the US Army because of the special application of piezoelectric resonators in missile guidance and control. His work during the 1980s–1990s using his first order perturbation theory achieved three orders of magnitude reduction in surface acoustic wave device acceleration sensitivity, which led to improved oscillators used by Raytheon for Patriot missiles and general improvement in commercial capability to current $10^{-10}/g$ level. Some of his results on resonator frequency stability are summarized in Ref. [19].

IX. BOOKS

Professor Tiersten coauthored the IEEE Standard on Piezoelectricity^[20] and was responsible for the theoretical part. This is the internationally most widely used reference on piezoelectricity. Tiersten was not interested in writing books and was satisfied by publishing his work as papers in journals. His 1969 book^[21] is one of the few classical books on piezoelectricity and has been a major reference on piezoelectricity for a long time. In 1990 he published another book^[22] on the development of basic equations in a polarizable and magnetizable continua. In Ref. [22] he used the cavity definition of fields in matter, which raises some questions on theories involving electric quadrupoles and higher-order moments.

X. CONCLUSIONS

Tiersten's work on continuum mechanics shaped the field many researchers are working in today. Some of the topics for which he had laid down the theoretical foundations are yet to

be further explored. His style is exemplary of Mindlin's school of applied mechanics, ranging from fundamental theories to applications in technology. There is no exaggeration to say that the loss of Professor Tiersten represents the end of an era of the theories of continuum electrodynamics. The marks left by him in this field are permanent.

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