Chapter 9

Measuring and marking counterparty risk

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Introduction

The volume of outstanding OTC derivatives has grown exponentially over the past 15 years. Market surveys conducted by the International Swaps and Derivatives Association (ISDA) show notional amounts of outstanding interest rate and currency swaps reaching US$866 billion in 1987, US$17.7 trillion in 1995, and US$99.8 trillion in 2002; an astonishing compounded growth rate of 37.2% per year.[1] Derivatives have expanded the opportunities to transfer risks, allowing for substantially improved risk sharing. They have also created connections among markets and market participants that are only just starting to be understood.

Counterparty risk, an example of one such connection, is the risk that a party to an OTC derivatives contract may fail to perform on its contractual obligations, causing losses to the other party. Losses are usually quantified in terms of the replacement cost of the defaulted derivatives and include, beyond mid-market values, the potential market impact of large and/or illiquid positions. Counterparty risks are bilateral – ie, both parties may face exposures depending on the value of the positions they hold against each other. Counterparty risks have mushroomed in the financial markets due to (a) the usual practice of 'offsetting' rather than 'unwinding' derivative positions and (b) the array of inter-dealer trades required to connect final risk-takers.

OTC derivatives and counterparty risks are focal points for market participants, policy-makers, regulators, accountants, tax authorities and many others. This chapter is an overview of the key issues relating to the measurement and pricing of counterparty risks.

Counterparty exposures

Definitions

- *Counterparty exposure* is the larger of zero and the market value of the portfolio of derivative positions with a counterparty that would be lost if the counterparty were to default and there were zero recovery. Counterparty exposures created by OTC derivatives are usually only a small fraction of the total notional amount of trades with a counterparty.
- *Current exposure* (CE) is the current value of the exposure to a counterparty.
- *Potential future exposure* (PFE) is the maximum amount of exposure expected to occur on a future date with a high degree of statistical confidence. For example, the 95% PFE is the

level of potential exposure that is exceeded with only 5% probability. The curve PFE(*t*), as *t* varies over future dates, is the potential exposure profile, up to the final maturity of the portfolio of trades with the counterparty. PFE(*t*) is usually computed through simulation models: for each future date *t*, the value of the portfolio of trades with a counterparty is simulated. A high percentile of the distribution of exposures is selected to represent the PFE(*t*). The peak of PFE(*t*) over the life of the portfolio is referred to as maximum potential future exposure (MPFE). PFE(*t*) and MPFE are often compared with credit limits in the process of permissioning trades.

- *Expected exposure* (EE) is the average exposure on a future date. The curve of EE(*t*), as *t* varies over future dates, provides the expected exposure profile. The expected exposures are computed with the same models used for PFEs. The EE(*t*) curve is referred to as the 'credit-equivalent' or 'loan-equivalent' exposure curve, and is used for credit pricing and for the calculation of the economic capital of well-diversified portfolios of counterparties.
- *Expected positive exposure* (EPE) is the average EE(*t*) for *t* in a certain interval (for example, for *t* during a given year).
- *Right-way/wrong-way exposures* are exposures that are positively/negatively correlated with the credit quality of the counterparty. Those exposures may have lower/higher expected credit losses associated with them than would be the case without correlation. A company writing put options on its own stock creates wrong-way exposures for the buyer. An oil producer selling oil in a swap creates right-way exposures for the buyer.
- *Credit risk mitigants* are designed to reduce credit exposures. They include netting rights, collateral agreements, and early settlement provisions.

Examining these credit risk mitigants in more detail, legally enforceable netting agreements allow trades to be offset when determining the net payable amount upon the default of the counterparty. Without netting, the position of the non-defaulting party would be a loss of the full value of the out-of-the-money trades against a claim on the total value of the in-the-money trades. With netting, positives and negatives are added first to determine the net payment due. For example, if a counterparty holds a currency option written by its bank with a market value of 50, while the bank has an interest rate swap with the same counterparty

Exhibit 9.1

Example of one path of counterparty's exposures

	Time (years)					
	0	*1*	*2*	*3*	*4*	*5*
Trade 001	–0.9	–1.4	0.5	0.1	–0.8	0.5
Trade 002	–0.4	–0.1	0.1	–1.4	–2.7	–3.2
Trade 003	1.1	2.0	1.7	1.2	1.4	0.8
Trade 004	–0.4	1.6	1.4	1.9	3.5	2.7
Trade 005	0.6	1.8	1.1	1.3	1.5	1.4
Exposure (US$)						
Without netting	1.7	5.4	4.7	4.4	6.4	5.4
With netting	0.0	3.9	4.7	3.1	2.8	2.2

Source: Authors' own.

having a mark to market value in favor of the bank of 80, then the exposure of the bank to the counterparty, with netting, is 30.

Exhibit 9.1 shows an example of the time paths of the values of five trades as well as the future exposures to the counterparty, with and without netting.

Collateral agreements require counterparties to periodically mark to market their positions and to provide collateral (that is, to transfer the ownership of assets) to each other as exposures exceed pre-established thresholds. Usually the threshold schedule is a function of the credit ratings of the counterparties. Counterparties with high/low credit ratings have to post collateral if amounts due exceed high/low thresholds. Collateral agreements do not eliminate all counterparty risks: exposures may exist below the thresholds; market movements can increase the exposure between the time of the last collateral exchange and the time when default is determined and the trades are closed out; the collateral received/posted depreciates/appreciates in value during the close-out period.

Liquidity puts, credit triggers and other early termination provisions reduce credit exposures by shortening the effective maturities of trades. Liquidity puts give the parties the right to settle and terminate trades on pre-specified future dates. Credit triggers specify that trades must be settled if the credit rating of a party falls below pre-specified levels.

Some counterparties trade many financial products (for example, loans, interest rate swaps, bond repos, foreign exchange options or commodity swaps). The ability to apply cross-product netting is desirable for the reasons mentioned above. Legal issues on the enforceability of netting arise when a dealer books trades at various legal entities, possibly based in different jurisdictions.

PFE models: an overview

Over the last decade, derivatives dealers have invested large amounts of resources to build sophisticated PFE measurement systems. Those systems comprise:

- databases (trades, agreements, legal entities, legal opinions, collateral holdings, risk limits);
- Monte Carlo simulation engines;
- trade pricing calculators;
- exposure calculators; and
- reporting tools.

The estimation of PFEs requires sophisticated models to simulate exposures in market scenarios on various future dates. The specification of the PFE model and its ongoing validation are of paramount importance. In the following subsections alternative model specifications are discussed, and certain choices and trade-offs are highlighted.

Simulation engine

Different market instruments require the specification of different stochastic processes to characterize their evolution through time. Interest rates in developed economies are often modeled as normal or lognormal diffusion processes. When interest rates are low, the normal diffusion may be more appropriate. When interest rates are high, the lognormal diffusion may be more appropriate. Many variants are possible. Major foreign exchange rates are usually modeled as lognormal diffusions. This practice is in contrast with the modeling of emerging market foreign exchange rates, where significant jumps can occur. Jump-diffusion processes are generally employed to characterize the movements of the prices of emerging market or pegged currencies. The stochastic processes followed by the emerging market currencies may not be Markovian: the past history of the exchange rates affects the likelihood of a large devaluation. For example, if a currency has not been devalued for a long period of time, then it may be more likely to be devalued in the near future. Immediately after a devaluation, the likelihood of another would be reduced. The shorter the time horizons over which PFEs are computed, the larger is the importance of modeling jumps, where required (see Das and Sundaram, 1999). For longer horizons, the impact of jumps tends to dissipate and the process cannot be easily distinguished from diffusion. Thus, the modeler must consider the horizon at which the PFEs are computed. Although commodity and equity prices are usually modeled as lognormal diffusions, it may be necessary to model jumps for some less liquid commodities and equities.

Sometimes the risk factor to be simulated is not a single price, but rather a string of prices. Examples are interest rate curves and commodity forward curves. In these cases, the simulation model must be sufficiently elaborate to impose the proper arbitrage-free constraints and realism on the reshapings of the curve. Arbitrage-free, multi-factor, term structure models have been applied to simulate the evolution of interest rate and commodity curves.

Some prices exhibit mean-reversion. For example, very high and very low interest rates are unlikely to persist for long periods of time in well-functioning economies. Mean-reversion is especially important when generating long-dated exposures. If mean-reversion (or some other form of volatility compression) is not incorporated, then unrealistic levels of exposures could be estimated at long horizons.

The calibration of the parameters of the simulation models is an important step in model building. The future values of the market risk factors are fundamentally determined by the calibration scheme. Models calibrated to historical data tend to project future values based on the statistical regularities observed in the past. Models calibrated to market prices (such as forward curves and option-implied volatilities) tend to reflect forward-looking views. There are positive and negative aspects in each form of calibration. Historical calibration implies that the process generating future market behavior is the same that was observed in the past. The model may be slow to react to changes in market conditions and structure, even if a time-decay factor is used to over-weight more recent observations. On the other hand, market prices contain components that are not the result of market participants' expectations about the future (for example, risk premiums, carrying costs, and so on). The objective of the simulation model is to project as realistically as possible the future developments in the markets being simulated. In that sense, the models should operate under the real probability measure. The only justification to use the risk-neutral measure is that, to some extent, it contains the consensus expectations of market participants on future prices, volatilities, and so on. When the simulations are used for pricing, the risk-neutral probabilities have to be used (see the 'Market valuation of credit exposures' section later in this chapter).

The correlations among market risk factors are the most important determinants of the potential future exposures created by large and well-diversified portfolios of trades. Positively correlated risk positions tend to increase credit exposures.

Trade pricing

Once a future market scenario is generated, in order to calculate the exposure in that scenario, all trades with the counterparty must be priced. That task consumes large computational resources. Imagine a swap dealer's book with 100,000 positions, with average maturity of seven years, and 2,000 market scenarios generated every three months. One would need to perform 5.6 billion pricings. At, for example, 100 microseconds per pricing, this would correspond to 156 hours of CPU time. Clearly, a good amount of financial and software engineering is required. In the early history of risk measurement, some large dealers retained super-computers for the computation of the credit exposures generated by their portfolios of OTC derivatives. Others developed heavily engineered algorithms to economize on the computation time (incorporating price grids, sophisticated interpolation schemes in both time and space dimensions, and so on). More recently, distributed/parallel processing has been employed. Unnecessary pricing complexity should be avoided: one should avoid refining within the margin of error. This is particularly important for long-dated exposures: the volatilities, correlations and probabilistic assumptions used in constructing the long-dated scenarios are themselves surrounded by considerable uncertainty. Constructing highly accurate (and computationally intensive) pricing calculators often qualifies as refining within the margin of error.

The collection and integration of market data, trades, agreements, legal and other information required to compute PFEs demands access to various front- and back-office systems spread throughout organizations. The level of cooperation among various departments must be high. Indeed, the data-integration challenge is an important reason used to explain why various first-tier financial institutions have not succeeded in evolving their PFE models beyond simplistic add-on rules. In addition, there is a need to develop and implement reporting tools to deliver the resulting information to various levels of decision makers in a timely and clear fashion.

Exposure calculation

After all trades with a counterparty have been repriced at a scenario/date, exposures can be computed. There are two fundamental concepts for the calculation of exposures: netting and margin nodes. A netting node is a collection of trades that can be netted. A margin node is a collection of trades whose values should be added in order to determine the collateral to be posted or received. The portfolio of positions with a single counterparty may comprise multiple netting and margin nodes. The counterparty exposure is determined by:

1. calculating the exposure in each netting node;
2. adding all netting node exposures;
3. calculating the collateral posted/received for each margin node;
4. adding collateral posted/received; and
5. calculating the net exposure to the counterparty as (2) minus (4).

The calculation method above, (1)–(5), implies that a counterparty with one netting node, one coincident margin node, and a zero collateral threshold would generate zero exposure. Sophisticated PFE models recognize that collateral does not change hands continuously and instantaneously. These models allow for a period of time between an event causing default and the final closing out of trades. A typical, conservative, close-out period is two weeks.

Model validation and control

All the computer code underlying a PFE model is extensively tested during the implementation phase, and re-tested on an ongoing basis via regression tests. Code changes are documented, independently verified and approved before released to production. The inputs to the model are reconciled with alternative sources of the same information. Counterparty positions are verified against the official books and records of the firm. The outputs of the model are verified for reasonableness under many conceivable stress scenarios. Historical, static backtesting is feasible and sometimes applied. Ongoing, dynamic backtesting is more difficult due to changes in the positions with each counterparty over time. Credit risk managers (users of the PFE model) provide valuable feedback on the quality of the output. Over the last decade, the validation and control requirements on PFE models have evolved to standards comparable to front-office calculators. This was primarily the result of the use of the outputs of those models for the market valuation of credit risk and calculation/allocation of economic capital. Internal and external auditors as well as industry regulators apply rigorous standards in ascertaining that the implementation and control of a PFE model is sound.

Applications of exposure modeling

The most important uses of PFE models are:

- trade approvals against credit line limits;
- credit risk valuation; and
- economic and regulatory capital.

Credit officers set limits on PFE profiles. The limits tend to be wider for short terms and tighter for long terms. In the process of permissioning new trades, the PFE profile to a counterparty is re-computed including the new trades. The PFE profile is then compared with the limit schedule. PFE models also generate the inputs for credit risk valuation. When exposures are uncorrelated with the credit quality of the counterparty, the unconditional expected exposure profile is used for valuation.

Another application of PFE models is the calculation of economic capital to support the risk of a portfolio of counterparties. The variability of exposures and the possible concentrations on certain market risk factors increases the risk of the portfolio. Ideally, a dealer would compute the economic capital of the portfolio of counterparties by applying a full simulation model – that is, a model in which market and credit risk variables are simulated simultaneously. In practice, many dealers tend to use expected exposure profiles, sometimes grossed up by a multiplicative factor to proxy for the increased risk of variable exposures. Canabarro, Picoult, and Wilde (2003) have shown that the gross-up factor depends on the characteristics of the portfolio of counterparties, with typical values in the range 1 to 1.25. That is, for the purposes of calculating economic capital, expected exposure profiles should be multiplied by that factor in order to produce results equivalent to full simulation.

Market valuation of credit exposures

The credit valuation adjustment (CVA) of an OTC derivatives portfolio with a given counterparty is the market value of the credit risk due to any failure to perform on agreements with that counterparty.[2] This adjustment can be either positive or negative, depending on which of the two counterparties bears the larger burden to the other of exposure and of counterparty default likelihood.

More generally, the market value of credit risk to a given counterparty incorporates the valuation of credit risk associated with all positions with that counterparty, including loans to, and inventories of securities issued by, that counterparty. Accurate pricing of these credit risks is an important determinant of mark to market earnings, and is the first line of defense in credit risk management. For example, underestimates of the market valuation of the credit risk with a given counterparty may bias prices in favor of accumulating larger risks with that counterparty, and the potential of large hidden losses that may only come to light at unfavorable events. Thus, in order to provide proper incentives to traders, the pricing of counterparty credit risk should be accurate. Some banks have set up internal credit risk trading desks charged with responsibility for pricing, hedging, and bearing the credit risk components of all counterparty positions.

In practice, it is typical to view the credit adjustment on the portfolio of positions with a given counterparty as an adjustment to the mid-market valuation of that portfolio. For example, consider an off-market yen–U.S. dollar currency swap between A, the yen receiver, and B. Suppose the mid-market valuation of the swap to the yen receiver is 100. Suppose this mid-market valuation already includes an effective market value of two for default risk to the hypothetical yen receiver, net of the market value of the default risk to the hypothetical yen payer. Suppose the actual swap between A and B has a net market value of default risk to A of five, because the yen payer happens to be of lower than mid-market quality. Then the credit-risk adjustment is a downward adjustment of three, leaving a fair market value to A of 97. In many cases, however, the default risk portion of the mid-market valuation is small and ignored, causing the credit adjustment to mid-market valuation to coincide with the total measured market value of the credit risk of the OTC position.

Mid-market valuations of at-market interest rate swaps, for example, are based on hypothetical counterparties of equal and high (say, AA) quality, so the effects of default risk are essentially offsetting. They need not be precisely offsetting, as the fixed-rate payer has slightly different expected exposures than the floating-rate payer. For example, in an upward-sloping yield curve environment, we expect the exposure to the floating-rate receiver to grow over time, as the floating rate is expected to grow, in a risk-neutral sense. Even so, on a plain-vanilla U.S. dollar interest rate swap between AA counterparties, the net market value of counterparty default risk associated with mid-market valuations is often indeed negligible. This may not be the case for other OTC positions, especially those with large expected exposures to only one of the two counterparties, such as long-dated options and off-market swaps.

Risk premiums

Investors demand a reward for bearing risk. The reduction in market value of an OTC position due to the counterparty's credit risk is typically larger than the actuarial expected loss, discounted to present value. For example, suppose a short-term bond promising US$100,000 will default with an actual probability of 30%, and in the event of default, half of the value is lost. The actual default probability might be estimated, for example, by the Moody's-KMV EDF measure (see Kealhofer, 2003). The mean loss rate is therefore 15%, the product of the loss probability with the fraction lost given default. We will ignore the time value of money for this example, taking interest rates to be zero. Given the mean loss rate of 15%, the expected payoff of the bond is 85,000. Investors, however, would typically pay less than 85,000 for this defaultable bond. In order to give an investor an incentive to buy this risky bond rather than a default-free bond with a payoff of 85,000, the market price of the defaultable bond must normally be something less than its expected payoff. For example, the defaultable bond may have a market price of 80,000. This means that, for pricing and trading purposes, bond investors may act as though they are neutral to risk, but assign an artificially high mean loss rate of 20%. In this sense, 20% is called the 'risk-neutral mean loss rate'. The risk-neutral mean loss rate is a key input to bond pricing and other credit risk pricing applications.

In practice, one could estimate the risk-neutral mean loss rate on a bond, or some other exposure such as a swap, from the prices of corporate bonds issued by the same or similar firms. Corporate bond prices reflect the probability of default in the same risk-neutral sense. Annualized risk-neutral mean loss rates, on average over the life of an exposure, are roughly the same as the counterparty's credit spread – that is, the portion of the counterparty's bond yields that is due to credit risk. For example, suppose that a bond yield is 8%, but would have been 7.5% were it not for the risk of its default. Then the average risk-neutral mean loss rate is roughly 50 bps. One should not estimate the yield spread associated with credit risk by taking the difference between the corporate bond yield and the yield of a Treasury bond of the same maturity, say 7%, for the Treasury yield is depressed from corporate bond yields by other important factors, notably the state tax exemption for U.S. Treasury coupon income, and the superior liquidity of Treasuries. Recently, default swaps, a class of credit derivatives, have become an important source of information regarding risk-neutral mean loss rates. A default swap obligates the counterparty buying default protection to pay the default swap rate at periodic coupon dates until maturity or default of the underlying 'insured' bond or loan, whichever arrives first. If and when default arrives first, the seller of protection effectively pays the buyer of protection the difference between the face value and the market value of the underlying bond or loan.[3] The default swap rate for a particular corporate bond is approximately equal to the average risk-neutral mean loss rate on the underlying bond.

OTC derivatives such as swaps may have systematically different fractions of exposure lost at default than corporate bonds, although there is not much evidence available on this point. One must use reasonable modeling assumptions.

Mean* exposure times mean* loss rate

A typical method for bilateral credit adjustments when only one of the two counterparties has a credit exposure is to compute the market value of the credit risk by adding up the discounted risk-neutral mean default loss, period by period, over the life of the positions between these two counterparties. (Discussion of two-sided default risk, which arises, for example, with interest rate swaps, is deferred until later.)

The total market value $V(t)$ of default risk during the t-th future time period (say, a future calendar quarter) is computed by the following steps.

1. Calculate $EE^*(t)$, the risk-neutral expected exposure for period t, that is, the average of

exposures weighted by their risk-neutral probabilities, over all possible scenarios. (The distinction between risk-neutral and actual expectations is emphasized with an asterisk.) This is the risk-neutral expected market value that would be lost with default during that period, with no recovery, given the effect of all applicable netting, collateral, and other credit enhancements. This exposure measure, EE*(t), can be calculated, for example, from derivative pricing algorithms. The risk-neutral expected exposure EE*(t) may in some cases be significantly different than the expected exposure uncorrected for risk premiums, and this certainly seems to be the case with U.S. Libor interest rate swaps.[4]
2. Calculate the risk-neutral mean default loss rate L*(t) associated with the period, which is the product of the risk-neutral likelihood of default during the period and the risk-neutral mean fraction of exposure lost in the event of default.
3. Obtain C(t), the price of a default-free zero-coupon bond of maturity t.
4. Calculate $V(t) = EE^*(t) \times L^*(t) \times C(t)$.

This calculation of V(t) ignores the effect of correlation between exposure and interest rates. To correct for this, one could calculate the expected exposure EE*(t) under forward risk-neutral probabilities.[5] Additional adjustments for correlation due to wrong-way or right-way exposure effects can sometimes be substantial. For example, a wrong-way increase in risk-neutral mean loss rates of 40 bps per 100-bp change in Libor could double the credit-risk adjustment on an interest rate swap (see Duffie and Huang, 1996). The total market value of the credit risk is V(1) + V(2) +…+ V(T), where T is the final time period of exposure to that counterparty.

This risk-neutral mean-discounted loss calculation of credit risk market value adjustments is reasonable in many cases. By the early 1990s, credit risk adjustments of the above variety had been developed for interest rate swaps by Sorensen and Bollier.[6]

General Monte Carlo approach

Risk-neutral Monte Carlo simulation can be used to obtain an estimate of the market value of credit risk in bilateral OTC portfolios, allowing for default by either counterparty, for general types of derivatives and other positions such as loans and options, for netting and collateral agreements, and for correlation between default risk and changes in underlying market prices and interest rates.

The general approach is to begin by calculating the market value V(B) of all future potential losses to a particular Party A due to default by Counterparty B. The same algorithm can likewise be used to calculate the market value V(A) of losses to Counterparty B through default by Counterparty A. The difference V(B)–V(A) is the net market value of the default losses to Counterparty A.

The algorithm is described in simple illustrative terms, and not in a complete and computationally efficient manner, as follows.

1. Initiate a new, independently simulated scenario.
2. Simulate, for this scenario, date by date, the net exposure of Counterparty A to default by Counterparty B. At each date, this gives the market value that would be lost if B were to default at that date, with no recovery, with all applicable netting agreements in force, and net of all collateral. The enforceability of netting and the valuation of collateral can also

be simulated if uncertain.

3. Simulate, date by date, whether or not B defaults at that date, and whether A defaults at that date.
4. If, at a given date, Counterparty B defaults and Counterparty A has not already defaulted, then simulate the fraction of the net exposure, as obtained in Step 2, that is lost. This determines the losses to A in this scenario.
5. Simulate the path of short-term interest rates.
6. Discount to present market value, using the compounded short-term interest rates for this scenario, the losses to Counterparty A.
7. Return to Step 1, unless a sufficiently large number of scenarios have been run to obtain the approximate effect of the law of large numbers.
8. Average the results of Step 6, over all independently generated scenarios. This average is the estimate of the market value of default losses to Counterparty A due to default by Counterparty B.

The net total market value of the default risk to Counterparty A is V(B)–V(A), the market value of default losses to Counterparty A that are due to default by Counterparty B, as above, net of the market value V(A) of the losses to Counterparty B due to default by Counterparty A. This difference V(B)–V(A) can be positive or negative.

Example

For a simple example, we present the net present value of default losses to Counterparty A, the payer on a plain-vanilla interest rate swap, and Counterparty B, the receiver. The swap is a 10-year (semi-annual coupon) U.S. dollar Libor swap with a fixed rate of 7.6% that would be at-market for this example if the counterparties were default-free. The notional amount of the swap is 1,000,000. The column of Exhibit 9.2 labeled 'PV(X)' shows the estimated present value (in hundreds) of the exposure to the payer in the event of default by the receiver, based on a three-factor affine term structure model, fit to historical Libor swap and swaption rates during the 1990s, as described in Chapter 12 of Duffie and Singleton (2003), upon which this example is based. This is the same as the price of the payer swaption, to enter a swap at a fixed rate of 7.6%, the indicated coupon date *t* with maturity 10-*t* years later. The volatilities of the model are set to match the volatility of the six-month-into-two-year at-market swaption at 16.5%. The column of Exhibit 9.2 labeled 'PV(Y)' shows the present value of the exposure to the receiver (which is the same as the market value of the corresponding receiver swaption). Note that, despite the fact that the swap rate is the at-market rate (for default-free counterparties) and the present values of the exposures to the two counterparties are thus similar, they are not the same. Indeed, although the two counterparties are assumed to be of equal credit quality, each having a credit spread of 50 bps and the same risk-neutral mean fraction 50% of loss given default, we shall see that the net present value of default losses to the payer is positive, while the net present value of default losses to the receiver is negative.

A flat term structure of credit spreads of 50 bps and a 50% risk-neutral mean fraction lost given default implies a constant risk-neutral default intensity of 100 bps for each counterparty. We will assume for simplicity that they default independently of each other and of changes in interest rates. As indicated in the algorithm above, in order to price the default losses to the

Exhibit 9.2

Example of mark-to-market for default on swap

Coupon date t	*PV(X) (100s)*	*PV(Y)*	*Loss rate (bp)*	*PVLR*	*PVLP*
0.5	167	217	24.9	53.9	41.4
1	216	288	24.6	70.9	53.2
1.5	243	327	24.4	79.7	59.2
2	256	348	24.1	84.0	61.9
Years 2.5–9 omitted					
9.5	25	34	20.8	7.0	5.2
Total				1,079.6	790.6
Credit adjustment				289.0	–289.0

Source: Based on results from Duffie and Singleton (2003).

payer at a given coupon date *t* associated with first default by the receiver at that date, we want the probability p(*t*) of this event. It can be shown (see Duffie and Singleton, 2003) that the loss rate for each coupon date *t*, which is p(*t*) times the 50% mean loss given default, is as shown in Exhibit 9.2, in basis points. These loss rates are the same for payer and receiver given the symmetry of credit qualities.

Now, we can calculate the present value PVLP = PV(X) × (loss rate) of the default loss to the payer due to first default by the receiver at coupon date *t*, as shown in the last column of Exhibit 9.2, and likewise the present value PVLR = PV(Y) × (loss rate) of the default loss to the receiver due to first default by the payer at that date. Adding these over all coupon dates (some of which, for brevity, are not shown in Exhibit 9.2) leaves the total present value of default losses to the receiver due to first default by the payer of US$1,080. Similarly, the total present value of default losses to the payer due to first default by the receiver is US$791. The net reduction in market value to the receiver due to default losses is 1,080 – 791 = US$289. There is a net upward credit adjustment for the payer of US$289. Because the swap rate is at-market for default-free counterparties, the mark to market value of the swap for the payer is also US$289 (positive).

General remarks on credit adjustments

There is some scope for reasonable variation in the computation of the market value of default losses, even among models that have the same conceptual foundations and intent. These differences can arise from reasonable but different inputs (such as risk-neutral mean loss rates) and also from different model structures. This is an active area of research and development.

In many cases, a downward adjustment from mid-market value is appropriate. Upward credit adjustments can also be appropriate, whether or not the counterparty is of higher credit quality than the broker-dealer. In general, the higher the relative quality of the counterparty, the greater is the fair market value of a given derivative to the dealer (an obvious point).

For example, consider an interest rate swap between an outside customer Z, and one of two possible dealers, Gilt, rated AAA, and Silver, rated BBB. Suppose that Z, rated AA, wishes to pay the floating rate, and first calls Gilt, receiving a quote R for the fixed rate to be paid by Gilt, set so that there is no initial exchange of cash, meaning that the fair market value of this swap between Z and Gilt is zero.

Now, suppose Z calls the lower-quality dealer Silver in order to obtain an interest rate swap under which Z pays floating and Silver pays the same fixed rate R. They negotiate an upfront price P to be paid by Silver that is greater than zero because Z was willing to receive a price of zero under the same contractual terms when trading with the higher-quality dealer, Gilt. Thus, Z should demand some positive price P in compensation for bearing the comparably higher credit risk of Silver. This means that an upward adjustment in the market value of the swap to Silver, relative to the price of zero obtained by Gilt. Even holding constant the credit qualities of counterparties, the credit risk adjustment can change sign as changes in market prices reverse the direction of exposure.

When dealers (and other firms) issue bonds, they sell them to investors at a price that reflects their own credit quality. The lower their quality, the lower the price at which they are willing to issue their bonds, relative to those issued by higher-quality firms. The same principle applies to derivatives.

Conclusion

In this chapter, we have discussed techniques that are currently used to measure and price counterparty risks. This is a rapidly evolving area of risk management. The phenomenal expansion of the credit derivatives markets, together with the maturation of techniques used to price and hedge counterparty risks, has caused a fundamental change in the perception and management of those risks. ‘Liquid’ counterparty risks are now hedgeable, and should be managed and assessed in the same manner as other hedgeable risks.

The traditional potential-exposure framework discussed in the first part of this chapter takes a static buy-and-hold view of counterparty risks, without incorporating the possibility of dynamic hedging of liquid credit risks. As the liquidity and completeness of the credit derivatives market continues to grow, more and more counterparty risks will become hedgeable. Static potential exposure measures will be superseded by Value-at-Risk (VaR) and stress limits similar to those currently used for other liquid market risks (such as those inherent in interest rates and currencies). Some firms have adopted internal credit risk hedging via desks whose purpose is to provide credit protection, at a transfer price, to other desks within the firm and then to choose how much of the firm’s net exposure to each counterparty is to be traded to outside counterparties. More decentralized firms have desk-by-desk hedging of credit risks.

These developments will have a fundamental impact in the asset and liability management of financial institutions that deal in derivatives. A large part of the counterparty risk of these institutions will be transferred via dynamic hedging, hopefully improving the risk sharing among many economic agents and increasing market efficiency.

It seems crucial that regulators appreciate this transformation. In particular, the framework used to assess counterparty risk should evolve *pari passu* with the possibilities of risk management.

The process of transformation of the static, buy-and-hold perception of counterparty risk into dynamic hedging programs is irreversible.

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[1] See published surveys on ISDA's website, www.isda.org.

[2] This section includes excerpts from 'Expert Report of Darrell Duffie', United States Tax Court, Bank One Corporation, Versus Commissioner of Internal Revenue, Docket Numbers 5759-95, 5956-97. See also, 'Decision of The Court, 2 May 2003'.

[3] For details and variants, see Duffie (1999).

[4] For a numerical example, see Duffie and Singleton (2003), Chapter 12.

[5] For an introduction to forward-risk-neutral calculations, see, for example, Hull (2000).

[6] The work of Sorensen and Bollier appeared in *Financial Analysts Journal*, May–June 1994, pp. 23–33.

A market measure of counterparty risk index is provided by Credit Derivative Research (CDR). The index measures the average CDS spread of the largest 15 credit derivative counterparties and thus, it does not take into account distress dependence among counterparties. This is a key element to take into account if risks are to be estimated adequately. Nor does the CDR index take into account a measure of counterparty liability (i.e., the â€œexposureâ€).Â The probability that at least one FI fails to deliver, given that a specific FI failed to deliver corresponds to the probability set marked in the Venn diagram (Figure 3). This probability set corresponds to the union of conditional PoDs of all the FIs in the system, given that a specific FI failed to deliver.