Magic squares on mobile phone

(Exhibit from the book: *Investigatation in School Mathematics*)

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Abstract:

Investigation is not only a method for a teacher to use in teaching mathematics, but also for a student to use in learning the subject. We show how to construct a magic square using the numbers on a mobile phone with the help of investigation. We then consider how the same problem can be approached in the early years of elementary school giving rise to many related problems.

The book *Investigation in School Mathematics* [1], published in 2006, was written for teachers, didacticians and students – future teachers of mathematics. This book describes:

- several important problem solving strategies – especially investigative ones and explains what investigation in school mathematics is;
- examples of problems which can be solved using the Investigations approach;
- mathematical situations which can be approach through investigations.

Investigations, as described in [1], is a method of teaching and learning mathematics which permits students to enter and penetrate more deeply into the world of mathematics that most other teaching approaches fail to do. The book has a strong practical complexion. It also illustrates for teachers and students what an investigation is before placing them in the position of learners when they are forced to take a fresh and invigorating look at school mathematics from a new perspective. School mathematics abounds with problems which can be turned into investigations. But when and how to use them in the classroom as a teaching approach must be left in the hands of individual teachers.

One of the attractive features of investigating a problem is the potential for children of all ages and abilities to have access to the problem. This is achieved by modifying both the problem and the investigative approach in order that **every** child achieves a level of success and satisfaction.

We now show an interesting and well known problem from [1] which can be solved with the help of an investigative approach. This is suitable for older children.

Since ancient times magic squares have fascinated mathematicians. The most well known magic square is that constructed by Albrecht Dürer on copperplate in 1514. In this article we discuss one simple example of a magic

1	2	3
4	5	6
7	8	9

Figure 1: Mobile phone

square which assumes only knowledge of the decomposition of a number into 3 natural numbers. This has application when teaching children about addition and the decomposition of natural numbers, and when they practise and apply these two aspects of mathematics in more challenging situations.

The number keys on mobiles have this square arrangement (see Figure 1). We introduce the problem by asking:
What is the sum of the three numbers in

- *the second column?*
- *the second row?*
- *the main diagonal?*
- *the other diagonal?*

We find that the four sums are all 15 and that the sums of the other rows and columns are not equal to 15.

Problem 1: Try now to move one or more of the number keys, 1 to 9, so that the sum of each row, each column and each diagonal is 15.

A 3×3 square arrangement of nine numbers where the sum of each row, each column and each diagonal is the same, is called a magic square. The sum is called the magic number of the square. In the case of our square the magic number is 15.

Investigating the problem

• We can start by positioning the numbers 1 to 9 randomly in the 3×3 square. This experimental strategy often results in no solution. We call this trial and error.

• A more reliable and effective strategy involves correcting errors and using insight so improving the possibility of finding a solution. We call this trial and improvement.

Both of these strategies may or may not lead to a solution, but always to a better understanding of the structure of the problem, which in itself is likely to lead to other strategies.

One such strategy begins by asking if it is possible to make a magic square using the nine numbers 1 to 9, and if so what its magic number will be. Some children may even ask if there are different magic numbers for the

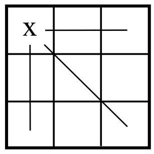

Figure 2: The contribution of a number

magic square using the same set of numbers 1 to 9. (We must always take great care not to assume that what we consider obvious is also obvious to children.)

The following questions guide children in the direction of answers to the two questions:

• *What is the total of the nine numbers, 1 to 9 ?* $(1 + 2 + \ldots + 9 = 45)$

• *As the nine numbers are shared equally between three rows (also three columns), what is the sum of the three numbers in each row?* $(45 : 3 = 15)$

This establishes that if a magic square exists, then its magic number must be 15.

We are now in a position to investigate where each of the numbers 1 to 9 should be positioned in the 3×3 square. Children sometimes need guidance in the form of questions to assist them with an investigation. Only one question should be asked at a time to allow children the opportunity to think about investigative strategies for answering a question.

• *Which four numbers must be in the four corner boxes?*

• *Which four numbers must be in the middle boxes along each side?*

• *Which number must be in the centre box?*

We consider the first question.
A number in any corner contributes to the sum of three decompositions one row, one column and one diagonal, as shown in Figure 2.

The magic number of our previously described square is 15. So we can search for all possible decompositions of 15 into three numbers which are chosen from $1, 2, 3, \ldots, 9$.

When 1 is one of the three numbers then we have only two decompositions.

$$15 = 1 + 5 + 9$$
$$15 = 1 + 6 + 8.$$

We have shown that for 1 to be a corner number it must be a member of at least three decompositions. Hence, 1 cannot be a corner number. Because a number in the middle of a side belongs to only one row and one column (two decompositions), then 1 must be in a middle square of a side.

When 2 is one of the three numbers then we have three decompositions.

2	9	4
7	5	3
6	1	8

Figure 3: Magic square

$$15 = 2 + 4 + 9$$
$$15 = 2 + 5 + 8$$
$$15 = 2 + 6 + 7.$$

Because a number in a corner square belongs to one row, one column and one diagonal, then 2 must be in a corner square.

When 5 is one of the three numbers then we have four decompositions.

$$15 = 5 + 1 + 9$$
$$15 = 5 + 2 + 8$$
$$15 = 5 + 3 + 7$$
$$15 = 5 + 4 + 6.$$

Because a number in the centre square belongs to one row, one column and two diagonals, then 5 must be in the centre square. Repeating the decompositions to include $3, 4, 5, \ldots, 9$ we can conclude that

$2, 4, 6, 8$ must be in corner squares;
$1, 3, 7, 9$ must be in middle of side squares;
5 must be in the centre square.

We can now put the numbers 1 to 9 into appropriate boxes in the 3×3 square and check the sums in rows, columns and diagonals to see if we have made a magic square. After several experiments using this approach a regular strategy can be seen for completing 3×3 magic squares. Figure 3 shows one possible magic square using the numbers 1 to 9.

Please check to see if this is true.

Question: Can you find any more solutions?

Answer: As 3×3 square has together 8 reflections and rotations (including the identity) we can construct, using Figure 3, seven more magic squares. We leave these for you to complete. Do you consider any of the eight solutions to be equivalent? How would you define equivalent in this situation?

Exercise 1: How many keys on the mobile (Figure 1) need to be moved to get anyone of the magic squares?

Answer: Only the number 5 remains in its original position so 8 of the keys need to be moved.

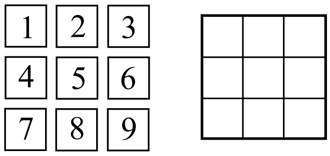

Figure 4: Number cards

Exercise 2: Given an empty 3×3 square with 5 in the centre box, how many numbers, including their positions, do you need to know to uniquely determine a magic square.

Answer: After experimenting you may discover that you need a minimum of 2 other numbers with the three following conditions:

1. they must not be in the same row, column or diagonal;
2. either the even numbers $2, 4, 6, 8$ should be in corner boxes and the odd numbers $1, 3, 7, 9$ should be in middle of side boxes;
3. the sum of the numbers is not equal to 10. (Why?)

Remark: Although a magic square arrangement on your mobile would be interesting to have it would not be very practical!

Our original problem can now be changed by using, for example:

- the numbers 2 to 10,
- any 9 even numbers,
- any 9 prime numbers.

For which of these sets of numbers do you think it is possible to make a 3×3 magic square?

What necessary and sufficient conditions does a set of 9 numbers have to satisfy in order to make a magic square?
We now look at other possible investigative approaches that you may wish to use with younger children with the original problem unchanged.

Children in the early years of elementary school find it helpful to have number cards 1 to 9 which they can move around from square to square using either a trial and error or, preferably, a trial and improvement strategy. Each child should have a set of the nine number cards and a large 3x3 grid as shown in Figure 4.

The material overcomes the difficulties created by the fixed positioning of a written number which children find inhibits their thinking and the mental operation of intuition. They respond positively to the freedom and openness of having such materials. However, one consequence of this approach is that children need to be taught the importance of recording both successful and unsuccessful trials. In a perfect world one would wish children to devise their

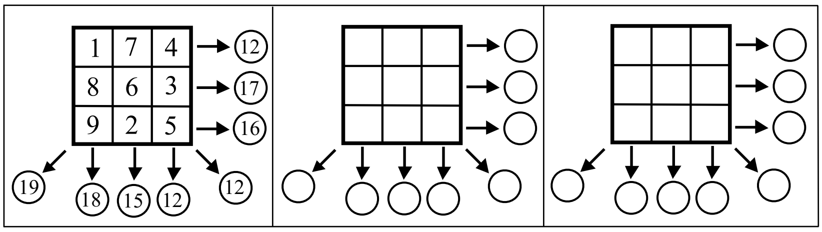

Figure 5: Recording sheet

own method of recording: this is possible with older children, but in the early learning stages of investigative work teachers should provide recording sheets. One such is described later.

We now look at changing the problem by removing some of the requirements of problem 1, in particular, not asking for rows, columns and diagonals to sum to 15. Instead, we allow children to place the nine numbers randomly in any of the squares. However, our ultimate target is that the children achieve the magic square with the magic number of 15. Let us explain the process by which this can be attained.

Problem 2: Put the nine numbers in the grid squares in any way you wish.

With every child having completed this we ask these questions:

- Who has a row/column/diagonal of three numbers with the greatest sum?
- Has anyone got two rows/columns/diagonals which have the same sum?

This becomes the next problem followed by successively more difficult problems each being an investigation in their own right.

Problem 3: Put your numbers in the squares so that two of your rows / columns / diagonals have the same sum.

Problem 4: Put your numbers in the squares so that all of your rows / columns / diagonals have the same sum.

Problem 5: Put your numbers in the squares so that every row, every column and every diagonal has the same sum.
The solution to problem 5 is in fact our magic square, but has been achieved without providing the information that the eight equal sums are 15, the magic number.

When working on Problems 2 to 5 children need a structured recording sheet. The one shown in Figure 5 works well with the circles used to record the sums. (Only the first row of the sheet is shown.)

Throughout children's schooling they are asked to solve or investigate problems that have been devised by teachers or appear in textbooks. We should look to change this with children posing their own problems, often arising out of ones they have investigated. Here are some you will find children suggest. You may like to investigate each one. We cannot guarantee that each has a solution. Perhaps you can prove that they do or do not.

Problem 6: Put your numbers in the squares so that every row, every column and every diagonal has an even sum.

Problem 7: Put your numbers in the squares so that the eight sums in the rows, columns and diagonals are consecutive numbers.

Problem 8: Put your numbers in the squares so that the eight sums in the rows, columns and diagonals form an additive sequence.

Most mathematical theories have both an experimental and inductive character. Their beginnings arise out of tentative searching and speculative trial and error; they gain a deductive character only after they a period of investigation. If we wish our students to have experiences of how mathematics evolves, then we should respect how mathematical theories come into existence, how they develop, and how they finally gain their formal nature. Too frequently students are only exposed to mathematics in its final and approved form. Using Investigations is one method of teaching that can substantially contribute to students being involved in the full range of the development of a mathematical theory. Investigations also provide students with insights into what it is like to be a mathematician and to experience mathematical thinking at work.

References:

[1] Orton, A., Frobisher, L.: Insights into Teaching Mathematics, London, Cassell, 1996

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[3] Trenkler, M: Magické štvorce a kocky – zdroj námetov na vyučovanie matematiky, *Disputationes Scientificae Universitatis Catholicae* 2(2002), č.1, 88-94

Download Mathematics Books for FREE. All formats available for PC, Mac, eBook Readers and other mobile devices. Large selection and many more categories to choose from.Â This book is intended to be used by children ages 5 to 6. Other age groups will also benefit from the book. Anyone can use this book globally, although the curriculum may differ slightly from one region to the other. This is so because the core content of Mathematics is the same around the world CBSE 12 & IIT-JEE Math Survival Guide-Definite Integrals. Mobile phones are permitted at school in the UK but pupils are not allowed to use them in class and they must be on silent during lessons. Teachers can take away phones if these rules are broken. School students can use their phones at break time and at lunchtime. Some teachers in British schools complain that pupils donâ€™t always follow the rules and that lessons are disrupted by people texting, making and receiving calls, looking at social networking sites, watching videos and even making videos in the class. â€˜My mum is scared that Iâ€™m going to lose my phone or that someone is going to steal