Review of *Matrix Analysis* by Roger A. Horn and Charles R. Johnson*

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The book is the first of two volumes; the companion volume will have the title *Topics in Matrix Analysis*. (There is an unfortunate misprint on the front cover. The second author is not Charles A. Johnson but Charles R. Johnson.)

The authors aim to present classical and recent results of matrix analysis that have proved to be important to applied mathematics. In its construction the book has some similarity with the now classic *A Survey of Matrix Theory and Matrix Inequalities* by M. Marcus and H. Minc, but it is broader in range and is organized for systematic study.

Many sections begin with motivations from concrete problems, and theorems are usually provided with full proofs. Many exercises as well as many problems (with occasional hints) of various degrees of difficulty, complement the exposition of the theory. Sometimes proofs of theorems are left to exercises or problems.

The book starts with a preliminary chapter, collecting not only necessary facts from elementary linear algebra but also some important facts like the Sylvester identity, the Binet-Cauchy formula, and the Schur determinantal formula.

Throughout the book, the authors present detailed study of classifications of matrices according to similarity, unitary similarity, *congruence, or ^{T}congruence, by establishing a canonical form (invariance) in each classification: Jordan form, inertia, rank, etc. In this connection, various canonical factorizations are discussed in detail: Schur unitary triangularization, UL factorization, QR factorization, singular value decomposition, polar decomposition, etc.

In a chapter devoted to Hermitian and symmetric matrices, among the main issues are variational characterizations of eigenvalues (the Courant-

*Cambridge U.P., Cambridge, England, 1985, xiii + 561 pp.

52 Vanderbilt Ave., New York, NY 10017 0024-3795/87/$3.50

Fischer theorem) and related eigenvalue inequalities. In connection with eigenvalue inequalities, a part of the theory of majorization is touched on, including the Birkoff theorem on doubly stochastic matrices in a later chapter. (The notation for majorization used here is the reverse of that in the monograph *Inequalities; Theory of Majorization and its Applications* by A. W. Marshall and I. Olkin.) A novel feature is the study of canonical diagonal reduction of a symmetric matrix via unitary ^{T}congruence (the Schur-Takagi theorem).

Positive definite matrices occupy one chapter. The order relation induced by positive semidefiniteness is treated rather briefly, whereas the study of classical eigenvalue and determinantal inequalities is well balanced; order invertivity of $A \to A^{-1}$ is proved, but not order preservation of $A \to A^{1/2}$. (Positive definiteness order is denoted by $\preccurlyeq$, while $\leqq$ is reserved for entrywise order.) The product of two positive definite matrices and its consequences are developed in some depth.

Norms on the space of matrices or vectors are viewed from the standpoint of error estimation. The norm of matrices as linear maps on a vector space with a suitable norm is characterized as a minimal submultiplicative norm (the Ljubič theorem). Unitarily invariant norms and symmetric gauge functions are mentioned briefly.

Theorems of Geršgorin type on eigenvalue location are developed in detail, including the results of A. Ostrowski, A. Brauer, and R. Brualdi. A result of a different character is the Hoffman-Wielandt theorem that for normal matrices A, B there is a permutation σ such that

$$\sum \left|\lambda_j(A) - \lambda_{\sigma_j}(B)\right|^2 \leqslant \|A - B\|_2^2,$$

$\|\cdot\|_2$ denoting the Frobenius norm. Its spectral norm version for Hermitian matrices (the Lidskii theorem) is not included.

In a chapter devoted to (entrywise) nonnegative matrices, a full account of the Perron-Frobenius theory is presented, including primitivity and asymptotic behavior of powers.

The references at the end of the book give a rather complete list of English-language monographs on matrices; the only foreign-language entry is the German translation of "Oszillationsmatrizen, Oszillationskerne und kleine Schwingungen mechanischer Systeme" by F. R. Gantmacher and M. G. Krein. One regrettable omission is the English translation of *Finite-dimensional Linear Analysis* by I. M. Glazman and Ju. I. Ljubič. The reviewer would like to take this opportunity to point out the recent appearance of a Russian book *Norms of Matrices and their Applications* by G. R. Belitskii and Ju. I. Ljubič, Kiev, 1984.

On the whole, the authors have done an excellent job of supplying linear algebraists and applied mathematicians with a well-organized comprehensive survey, which can serve both as a text and as a reference. The reviewer recommends that everyone in these fields have this book on his/her desk. One waits eagerly for the appearance of the promised companion volume, which will contain the following chapters: field of values, stable matrices and inertia, singular value inequalities, matrix equations and Kronecker products, matrices and functions, and totally positive matrices.

Received 29 September 1986; revised 2 October 1986

"The second edition of Matrix Analysis, as curated by Roger Horn and Charlie Johnson, is the definitive source and indispensable reference for the foundations of matrix analysis. The material is comprehensive yet thoughtfully collected, and presented with insightful exposition and crystal-clear organization.Â Charles R. Johnson is a Professor in the Department of Mathematics at the College of William and Mary. He is co-author of Topics in Matrix Analysis (Cambridge University Press, 1994). Read more. Roger A. Horn, Charles R. Johnson. Building on the foundations of its predecessor volume, Matrix Analysis, this book treats in detail several topics in matrix theory not included in the previous volume, but with important applications and of special mathematical interest. As with the previous volume, the authors assume a background knowledge of elementary linear algebra and rudimentary analytical concepts. Many examples and exercises of varying difficulty are included. show more.