Differential Geometry of Moving Surfaces, Solitons, and Filaments

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The purpose of these lectures is to present a unitary vision on the differential geometry of moving curves and surfaces, and some of their relationships to non-linear Integrable systems [1]. We begin by discussing the importance of the compact topology in representation theories. Then, we review the Poincare and Stokes theorems in a general cohomology language [2]. Next, we return to three-dimensional Euclidean spaces and introduce surface differential operators [3]. We will provide examples from flow theory, solitons on compact surfaces, and associated field theories [4]. The lectures will develop into a presentation of recent topological and geometrical results in Integrable curve dynamics and evolution [5], and the theory of motions of surfaces. More applications in fluid dynamics, compact nonlinear patterns [6], dynamics of vortex solitons [7], low dimensional systems [8], and vortices in mesoscopic superconductors [9] will be presented. The last topic will be related to applications of these structures in the dynamics and swimming of cells, flagella and cilia [10].

In the end, we will discuss future trends and mathematical open problems connected to the topic of moving compact boundaries, and possible applications in nanoscience, space physics and health sciences.

References

[1] Ludu A., *Nonlinear Waves and Solitons on Contours and Closed Surfaces*, Springer, Berlin, 2007.

[2] Prerequisites could be the basic ideas from C. Nash and S. Sen's book on topology and geometry, or M. do Carmo's book on classical differntial geometry. For algebraic topology we recommend W. Massey' s course. For vortex theory see H. Hasimoto, J. Fluid Mechanics **51** (1972) 477-, and a good starting point for the motion of surfaces is the paper by R. McLachlan and H. Segur in arXiv:solv-int/9306003 (1993).

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[4] Hou T.-Y., Lowengrub J. and Shelley M., Phys. Fluids **9** (1997) 1933.

[5] Calini A. and Ivey T., Journal of Nonlinear Science **15** (2005) 321–361; L. C. Garcia de Andrade, arXiv:astro:ph/0509847 (2005).

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Solitons and the Classical Differential Geometry of Surfaces in R3. Most expositions of soliton theory outline the his-. tory of the Korteveg de Vries (KdV) equation, be-. ginning with the physical observation of S. Russell.Â The Fundamental Theorem of Surfaces gives us a local correspondence between solutions of SGE and surfaces of constant Gaussian curvature âˆ'1 in R3 up to rigid motions. Although SGE has many global solutions defined on R2 , the corresponding surfaces always have singularities. In fact, Hilbert proved that there is no complete immersed surface in R3 with sectional curvature âˆ'1.